2.3 The product and quotient and higher order derivatives

Objectives: define and use basic rules of differentiation; define and calculate higher order derivatives

Finishing last lesson with a differentiability typical MC

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

The product rule:

Examples:

$h(x) = (3x - 2x^2)(5 + 4x)$

$h(x) = (x^3 - 3x)(2x^2 + 3x + 5)$

Example with data: Suppose h(x)=f(x)g(x).
Find h'(1) if f(1) = 2, f '(1)=-3, g(1)=5, and g'(1)=7.

ex. Suppose h(x)=f(x)g(x). Find h'(2). Look at picture.

The quotient Rule

Examples: $\frac{d}{dx}(\frac{5x-2}{x^2+1})$ $g(t)=\frac{t+1}{t^2+2t+2}$

Not every quotient needs the quotient rule: Look for…

$y=\frac{x^2+3x}{6}$ $f(x)=\frac{-3(3x-2x^2)}{7x}$

Data example. Suppose h(x)=f(x)/g(x).
Find h'(1) if f(1) = 2, f '(1)=-3, g(1)=5, and g'(1)=7.

Find all values where f has a horizontal tangent $f(x)=\frac{2x-1}{x^2}$

Practice: $\frac{d}{dx}\left[x^2\sin x\right]$ $\frac{d}{dx}\left[\frac{\cos x}{x^5}\right]$

$$\frac{d}{dx}\left[\frac{\cos x}{1-\sin x}\right]$$

<u>Let's derive Tangent using the quotient rule:</u>

<u>The other 3 derivatives:</u>

Find an equation of the line tangent to *s(t)* at *t*=2:

$$s(t)=\frac{t-1}{t+1}$$

<u>Higher Order Derivatives:</u> the derivative of the derivative is called the second derivative. The derivative of the second derivative is the third derivative.
Notation for higher order derivatives:

Example. Find the second and third derivative. $f(x)=x^3-4x+5$

<u>Acceleration:</u>

Average Acceleration: **Instantaneous:**

<u>Don't be an f'''(x)</u>

Example. Given $s(t) = 170t + 16t^3$

Find the average velocity on the interval [0,6]

Find the instantaneous velocity at t=2.

Find the average acceleration on the interval [0,6]

Find the instantaneous acceleration at t=2.

Example. Suppose the amount of oil, measured in gallons, in a tank at t minutes is given by

$v(t) = 3t^2 - 24t + 16 \text{ for } t \geq 0$

Find and include appropriate units of measure.

Is the amount of oil in the tank increasing or decreasing at t = 2?

Is the amount of oil increasing or decreasing at t = 5?

Is the volume of oil changing faster at t = 2 or t = 5?

When is the volume of the tank constant?

Example. The oil leaking from a tanker is expanding in a circular pattern. Find how fast the area of the spilled oil is changing when the radius of the spilled oil is 20 feet?